MODELING PIEZORESISTIVITY IN SILICON AND POLYSILICON

Gary K. Johns
Mechanical Engineering Department
Brigham Young University
Provo,Utah 84602

ABSTRACT

The piezoresistive effect of silicon is often utilized in sensors. Due to the crystalline nature of silicon, the sensitivity of the piezoresistive effect depends on many things including the direction and magnitude of the applied stress and the orientation of the crystallographic plane. A method of modeling the piezoresistive effect in silicon and poly crystalline silicon is presented. This mathematical model includes partial derivatives and linear algebra to combine basic electrical and mechanical equations to describe the change in resistivity as a function of stress. An experiment was used to verify the model. The analytical solution and experimental results agree. The model presented in this paper is adequate to predict the behavior of a piezoresistive sensor under uniform stress.

NOMENCLATURE

$$E_i = Electric\ Field$$
$$J_i = Current$$
$$\rho = Resistivity$$
$$\sigma_i = Stress$$
$$\tau_i = Shear\ Stress$$
$$R = Electrical\ Resistance$$
$$L = Length$$
$$A = Cross\ Sectional\ Area$$
$$l_i, m_i, n_i = Directional\ Cosines$$
$$\varepsilon = Strain$$
$$\pi_{ij} = Piezoresistive\ Coefficients$$

INTRODUCTION

Piezoresistivity is a material property that couples bulk electrical resistivity to mechanical strain [1]. In other words, as a material is stretched, compressed, or distorted in any way, the electrical resistivity changes. Almost all materials have this property, even though in some it is so small that it is virtually undetectable. However, other materials show a large change in resistivity for a relatively small strain. One of these materials is silicon. The piezoresistive effect in silicon makes it a good candidate to use as a sensor. A sensor element is a device that converts one form of energy into another [2]. An element made from silicon can be designed to be a sensor since the piezoresistive property links strain energy to change in resistance (electrical energy). This change in resistance can be related to a physical phenomenon such as displacement, pressure, or force. A mathematical model of the piezoresistive effect will be presented, which may be used to design piezoresistive sensors in silicon or polysilicon.

BACKGROUND

In 1856, William Thomson, more commonly known as Lord Kelvin, first discovered the piezoresistive effect when he noticed that the electrical resistance of copper and iron wires changed when subjected to a mechanical strain. This phenomenon became known as piezoresistivity: "piezo" meaning "to push" in Greek, and "resistivity" from the change in electrical resistance. In 1954, Charles Smith, working for Bell Laboratories, published a paper [3] exposing the piezoresistive properties of silicon and germanium. He set up experiments in order to derive a method of modeling this effect; from which he derived piezoresistive coefficients that relate change in resistance to stress. His experimental setup is illustrated in Figure 1. He also found that silicon and

germanium are much more sensitive to the piezoresistive effect than most metals. In the 1960's, silicon began to be used as a sensor on thin membranes by diffusing it on areas of the membrane that would experience the most stress in order to measure pressure. It was also doped on cantilever beams to enable measurement of other phenomena (see Figure 2). Since then, many sensors have been developed that are based on the piezoresistive effect of silicon.
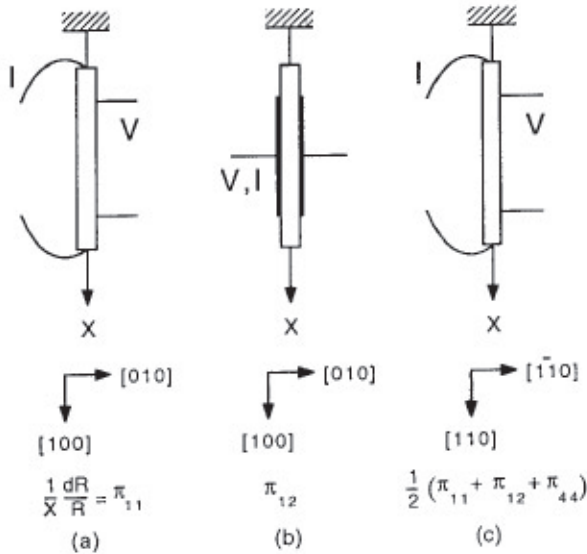

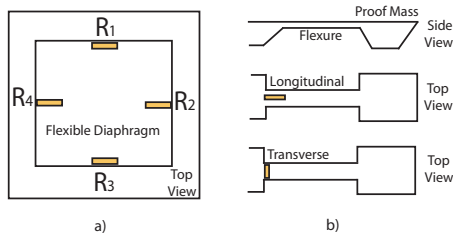Figure 1. Charles Smith's experimental setup

Figure 2. Piezoresistive sensor elements doped onto diaphragms and beams

MODELING PIEZORESISTIVITY ALONG CRYSTAL AXES

Modeling the piezoresistive effect of silicon requires linking equations related to the electrical properties of a material with equations describing the stress or strain and volumetric changes of a material. Starting with Ohm's law ($E = IR$), the electric field vector of an anisotropic cubic crystal can be derived. It is related to the current vector by a resistivity tensor [4].

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_6 & \rho_5 \\ \rho_6 & \rho_2 & \rho_4 \\ \rho_5 & \rho_4 & \rho_3 \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} \tag{1}$$

The resistivity tensor is symmetric and is therefore made up of only six components. When the crystal is in an unstressed state, the resistivity components along the diagonal of the matrix, which represent the resistivity along the <100> axes of the cubic, are equal to each other. The other three components are also equal to each other and are 0.

$$\rho_1 = \rho_2 = \rho_3 = \rho, \qquad \rho_4 = \rho_5 = \rho_6 = 0 \tag{2}$$

When the cubic is stressed, each component of resistivity changes. The new resistivity of each component in the resistivity tensor is

$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \end{bmatrix} = \begin{bmatrix} \rho \\ \rho \\ \rho \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \\ \Delta\rho_5 \\ \Delta\rho_6 \end{bmatrix} \tag{3}$$

The link relating change in resistance to stress is the Π matrix. This is the matrix which Charles Smith worked to define. The experimental setup in Figure 1 shows how he derived the piezoresistive coefficients for silicon and germanium. The change in resistance is directly related to the stress on the object through these piezoresistive coefficients by

$$\frac{1}{\rho}\Delta\rho_i = [\pi_{ij}][\sigma_j] \tag{4}$$

where the stresses relate to the stresses shown in Figure 3. σ_1, σ_2, and σ_3 are along the axes of the cube and σ_4, σ_5, and σ_6 map to the shear stresses τ_1, τ_2, and τ_3. With six stress components and six resistivity components, the π matrix must be a six-by-six matrix. Therefore, 36 π coefficients are required to populate the
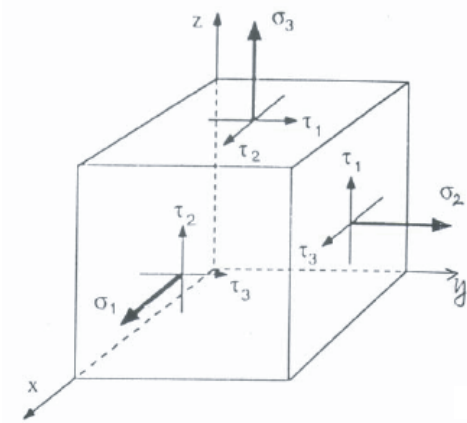
Figure 3. Stress Cube

matrix. However, for the cubic crystal structure of silicon, the matrix simplifies to

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{bmatrix} \tag{5}$$

Only three π coefficients (π_{11}, π_{12}, and π_{44}) are needed. When all of the values of the components are filled in, equation 4 becomes

$$\frac{1}{\rho}\begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \\ \Delta\rho_5 \\ \Delta\rho_6 \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{6}$$

As shown in [5], [1], and [4], equations 1, 3, and 6 can now be combined to obtain

$$\begin{aligned} E_1 &= \rho J_1 + \rho\pi_{11}\sigma_1 J_1 + \rho\pi_{12}\left(\sigma_2 + \sigma_3\right)J_1 + \rho\pi_{44}\left(J_2\tau_3 + J_3\tau_2\right) \\ E_2 &= \rho J_2 + \rho\pi_{11}\sigma_2 J_2 + \rho\pi_{12}\left(\sigma_1 + \sigma_3\right)J_2 + \rho\pi_{44}\left(J_1\tau_3 + J_3\tau_1\right) \\ E_3 &= \rho J_3 + \rho\pi_{11}\sigma_3 J_3 + \rho\pi_{12}\left(\sigma_1 + \sigma_2\right)J_3 + \rho\pi_{44}\left(J_1\tau_2 + J_2\tau_1\right) \end{aligned} \tag{7}$$

which describe the electric field potential along the crystal axes. The first term in each of the equations is the unstressed condition of the cubic. The next term takes in the effect of the stress along the axis of the respecting electric field component. The effect of the other two axial stresses are presented in the third term, and the last term models how the shear stresses effect the electrical potential.

MODELING PIEZORESISTIVITY IN ANY DIRECTION

The above equations are used to model the piezoresistive effect of silicon in the direction of the crystal axes. However, it is often convenient to be able to model the piezoresistive effect along other directions as well. Two new π coefficients can be defined in terms of the original 3 π coefficients and an arbitrary direction. These two new π coefficients relate the change in resistance to the stress in the longitudinal direction (defined to be the same direction as the current flow) and the transverse direction (defined to be perpendicular to the current flow). π_l and π_t are

$$\begin{aligned} \pi_l &= \pi_{11} + 2\left(\pi_{44} + \pi_{12} - \pi_{11}\right)\left(l_1^2 m_1^2 + l_1^2 n_1^2 + m_1^2 n_1^2\right) \\ \pi_t &= \pi_{12} - \left(\pi_{44} + \pi_{12} - \pi_{11}\right)\left(l_1^2 l_2^2 + m_1^2 m_2^2 + n_1^2 n_2^2\right) \end{aligned} \tag{8}$$

l_1, m_1, and n_1 are directional cosines between the longitudinal direction of the sample and the cubic axes and l_2, m_2, and n_2 are the directional cosines between the direction of one component of the transverse stress and the cubic axes [6].

LONGITUDINAL EXAMPLE OF PIEZORESISTIVITY

A simple example that demonstrates the modeling of the piezoresistive effect can be seen in the uniaxial stress case. The setup for the example is shown in Figure 4. The resistance of the
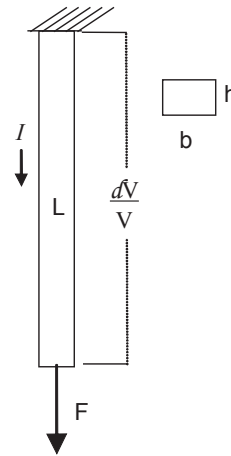

Figure 4. Tension example

unstressed beam, R, is given by

$$R = \frac{\rho L}{A} \tag{9}$$

3

where ρ is the bulk resistivity, L signifies length, and A is the cross sectional area. By implicit differentiation:

$$\delta R = \delta\rho\left(\frac{L}{A}\right) + \delta L\left(\frac{\rho}{A}\right) - \delta A\left(\frac{\rho L}{A^2}\right) \qquad (10)$$

Dividing equation 10 by the original resistance equation yields

$$\frac{\delta R}{R} = \frac{\delta\rho}{\rho} + \frac{\delta L}{L} - \frac{\delta A}{A} \qquad (11)$$

Equation 11 can be approximated as

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta L}{L} - \frac{\Delta A}{A} \qquad (12)$$

In this form, it is clear from the equation that a fractional change in resistance, $\frac{\Delta R}{R}$, is influenced by the fractional change in bulk resistivity, $\frac{\Delta\rho}{\rho}$, and by volumetric changes, $\frac{\Delta L}{L}$ - $\frac{\Delta A}{A}$, of the beam. Depending on the material, either the bulk resistivity change or the geometric change can be dominate the change in resistance. In silicon, the bulk resistivity change in usually the most dominant. Each of the components associated with the volumetric changes can be written in terms of strain.

$$\frac{\Delta L}{L} = \varepsilon_l, \qquad \frac{\Delta A}{A} = 2\varepsilon_t \qquad (13)$$

where ε_l denotes the strain along the length of the beam and ε_t is the strain across the cross section of the beam. The fractional change in area, $\frac{\Delta A}{A}$, is equal to the fractional change of the width plus the fractional change of the height, $\frac{\Delta w}{w} + \frac{\Delta h}{h}$, with strains $\varepsilon_w = \varepsilon_h = \varepsilon_t$. Using poison's ratio, ν, to relate ε_l and ε_t through the equation $\varepsilon_t = -\nu\varepsilon_l$ simplifies equation 12 to

$$\frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \varepsilon_l(1 + 2\nu) \qquad (14)$$

The change in bulk resistivity, $\frac{\Delta\rho}{\rho}$, gives an even larger contribution to the change of resistance. As seen above, Charles Smith's experiment was set up to relate stress to change in resistivity. He related these two phenomena through a Π matrix. Using that relationship as well as equation 8, the fractional change in resistance along the length of the beam becomes

E = Young's modulus

$$\left(\frac{\Delta R}{R}\right)_l = \varepsilon_l(\pi_l E + 1 + 2\nu) \qquad (15)$$

In a similar manner the fractional change in resistance in the transverse direction can be found to be

$$\left(\frac{\Delta R}{R}\right)_t = \varepsilon_t(\pi_t E - 1) \qquad (16)$$

MODELING PIEZORESISTIVITY IN POLY CRYSTALLINE SILICON

Many MEMS devices are made from polysilicon instead of pure silicon. The π coefficients used in the silicon Π matrix are no longer valid with polysilicon. Polysilicon can be approximated as several small single crystalline grains of silicon separated by grain boundaries [5]. The larger the grains, the more the crystalline material acts like single crystalline material. Burns, [5], estimates the π_l, and π_t coefficients for polysilicon by averaging equation 8 over all possible orientations and weighing each value by its probability of occurring. His results give the average of value of π_l and π_t in the following two equations.

$$\pi_l = \pi_{11} - 0.4(\pi_{11} - \pi_{12} - \pi_{44})$$
$$\pi_t = \pi_{12} + 0.133(\pi_{11} - \pi_{12} - \pi_{44}) \qquad (17)$$

RESULTS

The above equations were used to model the piezoresistive effect on pure tension in a beam. The beam was modeled on the micro level, and was a assumed to be made from n-type polysilicon. The original bulk resistance, modulus of elasticity, Poisson's ratio, and π coefficients were approximated from literature. The results showed a linear decrease in resistance with an increase in stress. An actual specimen was then fabricated and tested. A picture of the tensile beams and the force gauge used to measure stress is shown in Figure 5. Figure 6 shows the results from two tensile specimens and the analytical solution. The experimental results are seen to match very well the analytical prediction.

CONCLUSION

Piezoresistivity is a material property that can be utilized in sensors. Mathematical modeling of piezoresistivity aids design of sensors. However, in modeling piezoresistivity, it is important to know the crystal directions and crystal structure of the device being modeled. The results show a very close correlation between experimental results and analytical predictions. The equations presented above were mainly taken from literature. If more complex stress situations are required, the author suggests further research in the referenced material. Finite element modeling may also be used where complex geometries are involved.
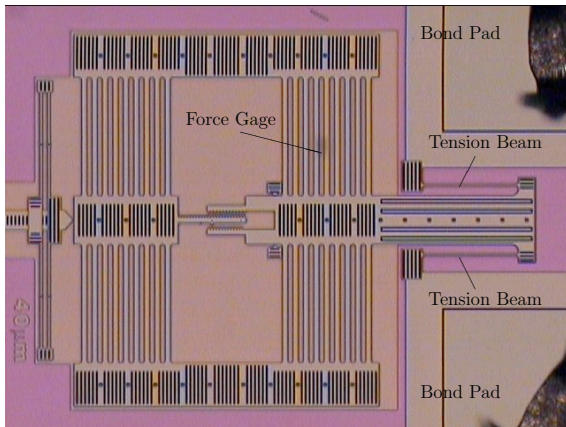
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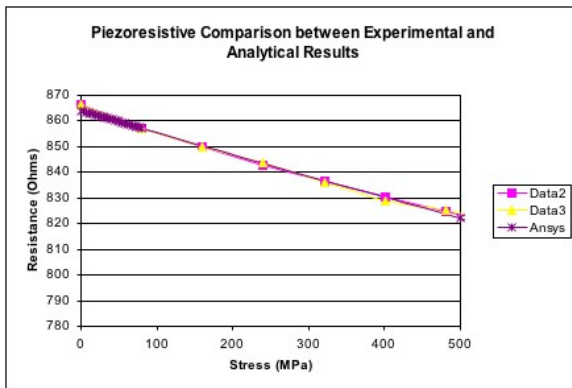Figure 5. SEM picture of a tensile specimen. Courtesy of Rob Messenger

Figure 6. Tension example

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